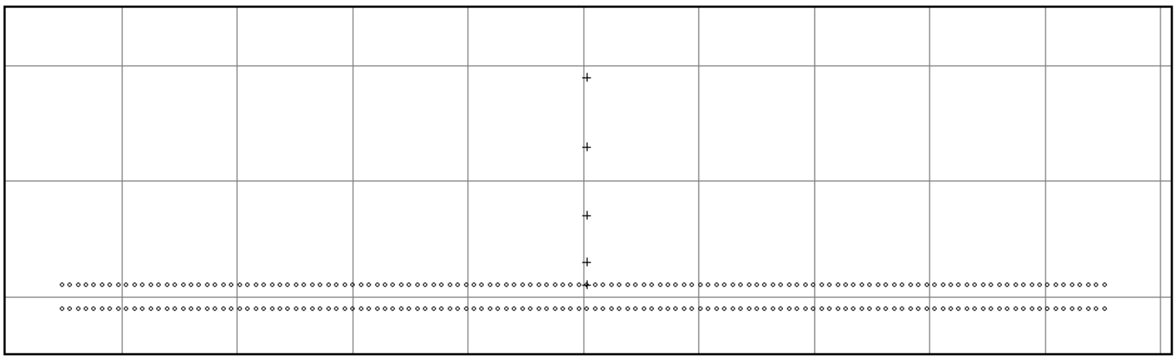
Durham Model Aquifer- Pumping test March 23, 2018

Analysis using MLU for Windows

General setup

A discussion in the LinkedIn group "Hydrogeology Forum" introduces the DMA pumping test. The aquifer is man-made and only 26 m by 0.7 m. Boundaries may be assumed no-flow. The aquifer is phreatic with a thickness of 1.5 m, of which 1 m saturated. The test lasted almost 2 hours and drawdown was measured in the well and four observation boreholes. At the end the measured drawdown in the pumping well was 0.62 m.

For the analysis of this pumping test the MLU software is used. Since MLU is based on an analytical radial flow solution, image wells are required to account for the no-flow boundaries. All wells (diameter 0.15 m) are projected in two rows that are 2 m apart. Each row consists of 90 wells, 0.7 m apart. Later on each row was extended to 130 wells as a check to see whether this improves the results. Since the results did not change, the number of image wells is large enough. In the plan view all wells are indicated by a small circle, the wells (BH1 up to BH5) by a cross and each square is 10-by-10 m.

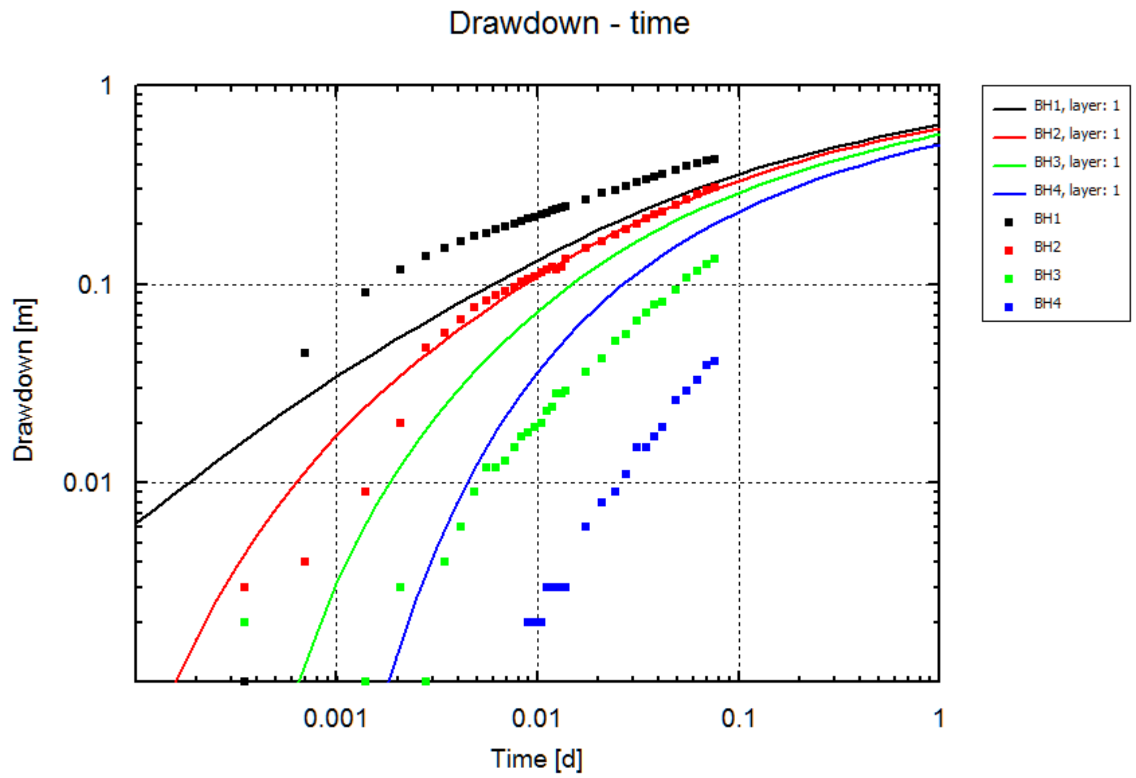


MLU does not allow for a reduced transmissivity when the water table is lowered during the test. Therefore Jacob's correction (Kruseman & de Ridder, page 101, 102) is applied to all measured drawdowns.

Step 1: Analysis using Theis

When using the Theis model, each individual observation well drawdown curve can be simulated. However, each model does only fit one obs. well at a time. Resulting T and S-values when individual obs. wells are analysed:

Theis individual obs.well analysis			
	T(m^2/d)	T(m^2/s)	S
BH1	414	0.0048	0.0081
BH2	337	0.0039	0.0318
BH3	330	0.0038	0.1123
BH4	574	0.0066	0.2331

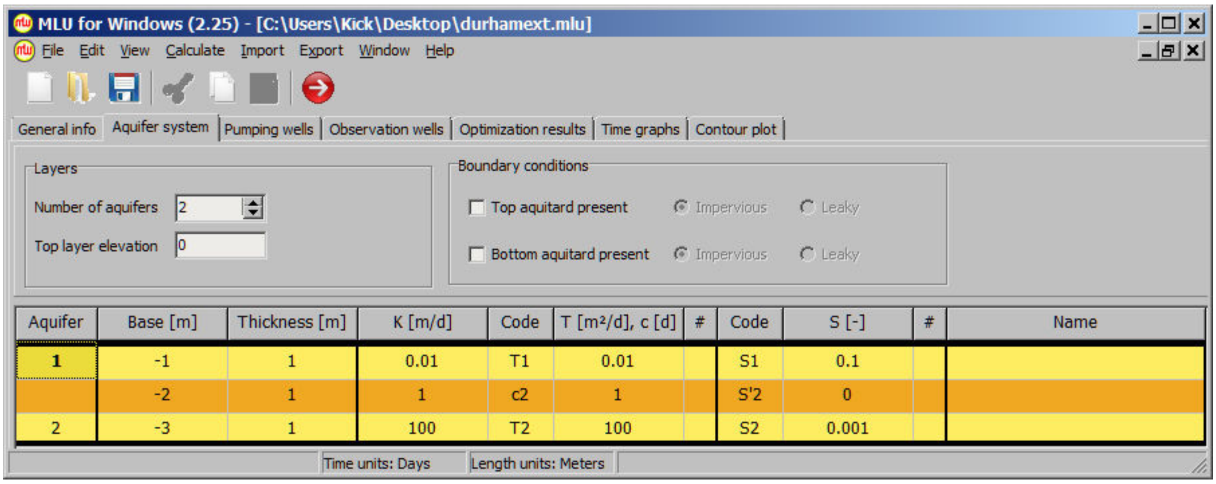


In above example T and S-values are found using drawdown data of BH2 only. There is no single set of T and S values in a Theis model that matches more than one obs. well. Also, when obs. wells are analysed with increasing distance to the pumping well, it appears that the resulting S-values also increase (see above table). This is an indication that Theis is a too simple model to analyse the measured drawdown data.

Step 2: Analysis using Boulton

More realistic models for pumping tests in water-table aquifers are Boulton and Neuman. For the present case with its very thin saturated aquifer (1 m) and fully penetrating pumping and observation wells, the vertical flow component within the aquifer is only small. Therefore the Boulton model is a more likely option. In MLU the delayed yield of a Boulton model is created by adding a very thin layer on top of the pumped aquifer, which represents the water table. This layer has a near-zero transmissivity and an S_y that differs from the aquifer S-value. The delayed yield (the flow between both layers) is subject to a certain resistance c (days).

In a Boulton model there are four unknown parameters: T, c, S and S_y. For example, when T=100 m^2/d, S=0.001, c=1 d and S_y=0.1, an MLU model looks like:



Aquifer	Base [m]	Thickness [m]	K [m/d]	Code	T [m²/d], c [d]	#	Code	S [-]	#	Name
1	-1	1	0.01	T1	0.01		S1	0.1		
	-2	1	1	c2	1		S'2	0		
2	-3	1	100	T2	100		S2	0.001		

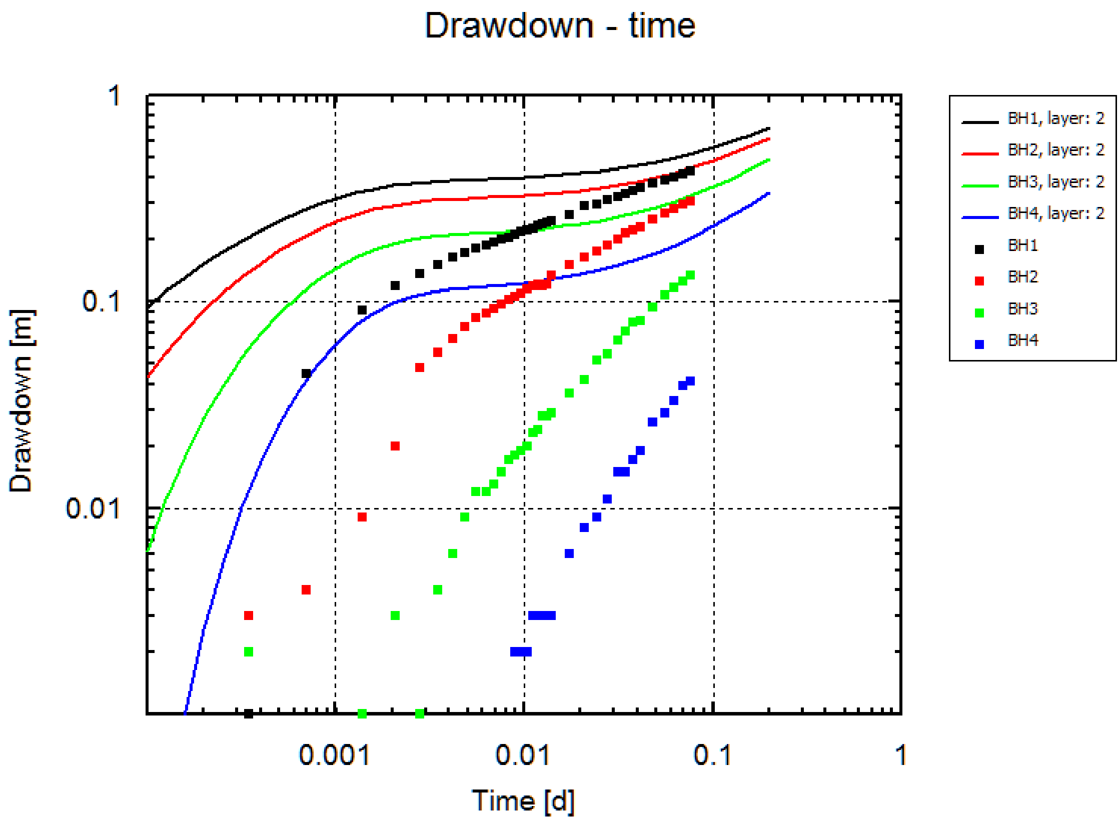
A symbol (character or digit) is added (in the # column) to each hydraulic properties that has to be optimized during the calibration process.

Aquifer	Base [m]	Thickness [m]	K [m/d]	Code	T [m²/d], c [d]	#	Code	S [-]	#	Name
1	-1	1	0.01	T1	0.01		S1	0.1	d	
	-2	1	1	c2	1	c	S'2	0		
2	-3	1	100	T2	100	a	S2	0.001	b	

Time units: Days | Length units: Meters

Parameter values are estimated in MLU by automated calibration (inverse modeling, nonlinear regression analysis). The optimization algorithm finds a “best fit” (minimized value for the SSE, Sum of Squared Errors) solution with associated statistical results.

Before optimization measured and computed drawdown do not match at all



During optimization, the following table is presented. It shows that the four parameters are found simultaneously in 9 iterations. During this process the Sum of Squared Errors (SSE) is reduced from 4.2585 m^2 to 0.0050812 m^2.

MLU progress and output results

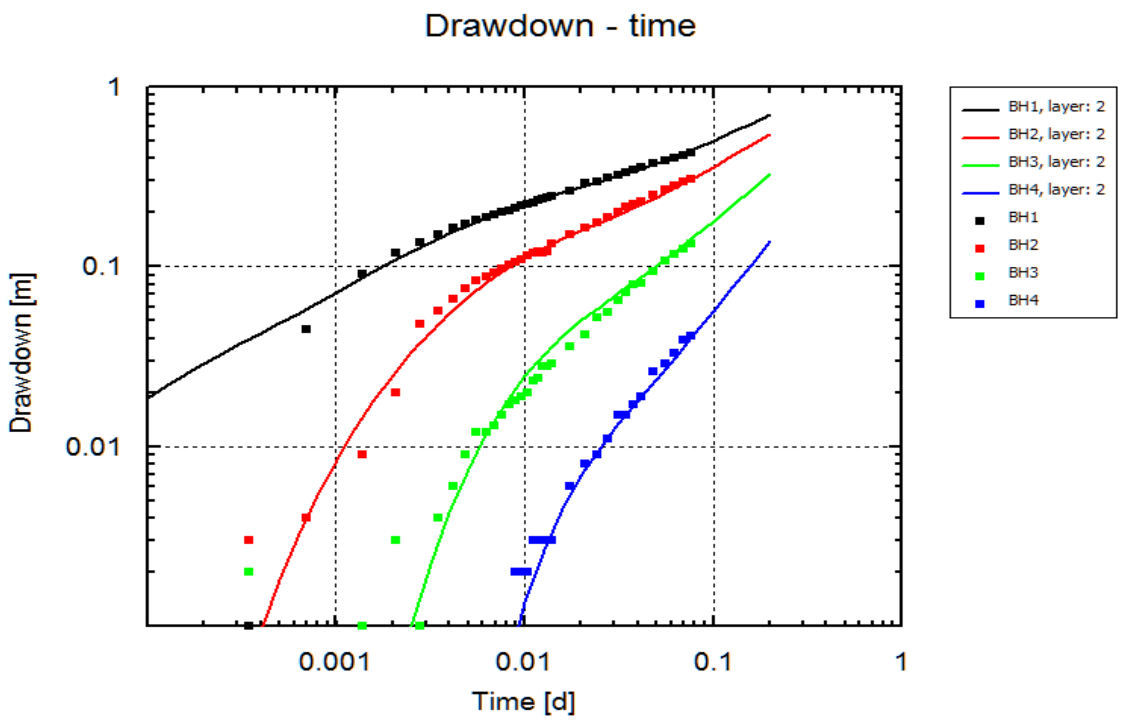
Number and Results of Successive Iterations

No.	T 2 [m²/d]	c 2 [d]	S 1 [-]	S 2 [-]	SSE [m²]
0	100.0	1.000E+00	1.000E-01	1.000E-03	4.2585E+00
1	8.933E+01	3.197E-01	9.456E-02	2.411E-03	5.0172E-01
2	6.150E+01	1.527E-01	1.207E-01	4.764E-03	6.6141E-02
3	5.008E+01	1.270E-01	1.241E-01	9.467E-03	3.0448E-02
4	4.901E+01	1.554E-01	1.195E-01	2.152E-02	1.7212E-02
5	4.869E+01	2.359E-01	1.188E-01	3.135E-02	7.1007E-03
6	4.850E+01	2.749E-01	1.304E-01	3.176E-02	5.1559E-03
7	4.845E+01	2.806E-01	1.338E-01	3.224E-02	5.0824E-03
8	4.845E+01	2.822E-01	1.342E-01	3.236E-02	5.0812E-03
9	4.845E+01	2.825E-01	1.343E-01	3.239E-02	5.0812E-03

parameters found

Accept result Reject result

After optimization (calibration) the model matches the data of all four observation wells.



Measured and computed drawdowns for four boreholes on a log-log scale

Aquifer	Base [m]	Thickness [m]	K [m/d]	Code	T [m²/d], c [d]	#	Code	S [-]	#	Name
1	-1	1	0.01	T1	0.01		S1	0.134303	d	
	-2	1	3.539254	c2	0.282545	c	S'2	0		
2	-3	1	48.44803	T2	48.44803	a	S2	0.032385	b	

Time units: Days Length units: Meters

THE CALCULATED LEAST SQUARES SOLUTION

Parameter value + Standard deviation

T 2	4.845E+01	+	6.254E-01	(1 %)
c 2	2.825E-01	+	1.019E-02	(4 %)
S 1	1.343E-01	+	3.406E-03	(3 %)
S 2	3.239E-02	+	8.979E-04	(3 %)

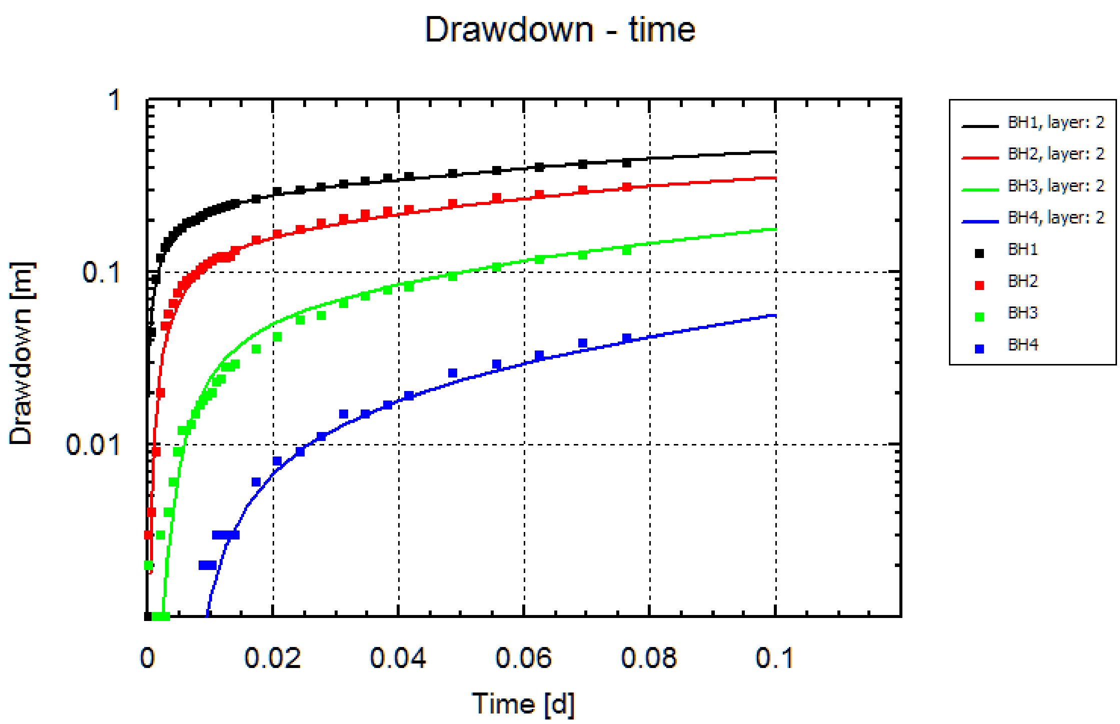
Initial sum of squares is	4.2585
Residual sum of squares is	0.0051
Improvement last iteration	2.8E-14
Number of iterations	9
Condition number	47.6

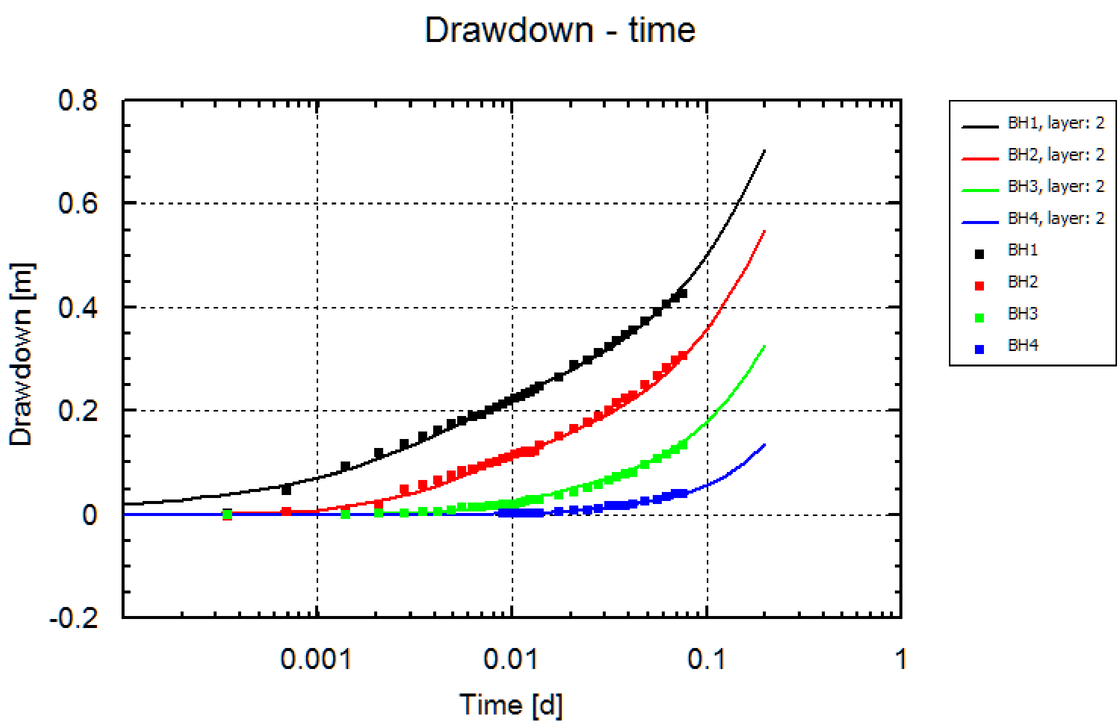
Correlation matrix (%)

T 2	100			
c 2	13	100		
S 1	-30	59	100	
S 2	-11	81	33	100

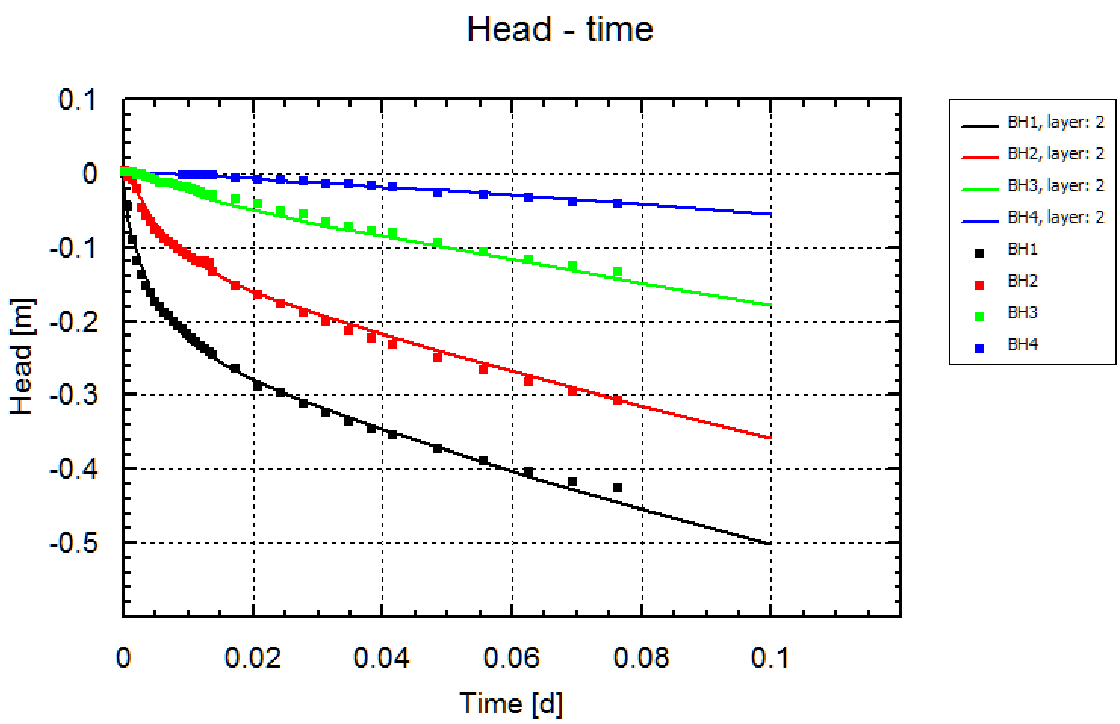
It appears that all four parameters can be obtained within small ranges.

T = 48 m^2/d, S = 0.032 and S_y=0.13

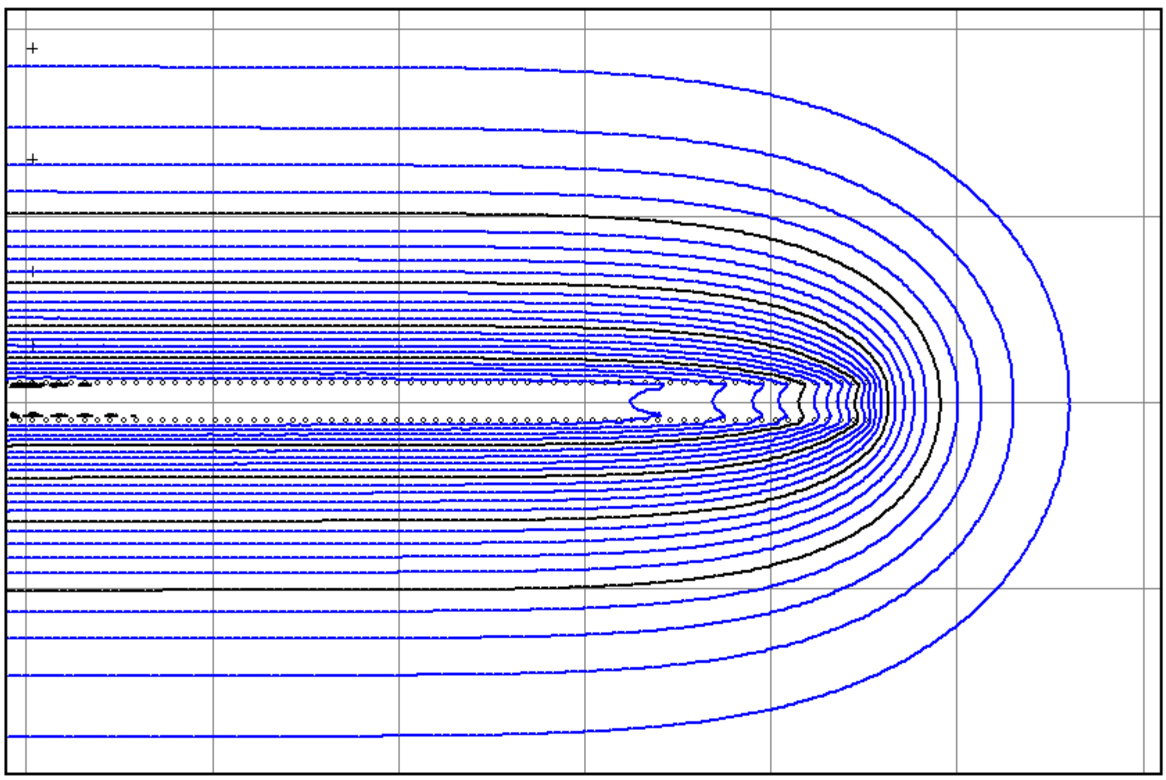




Measured and computed drawdowns on a a linear-log scale and log-linear scale



Measured and computed heads on a linear-linear scale

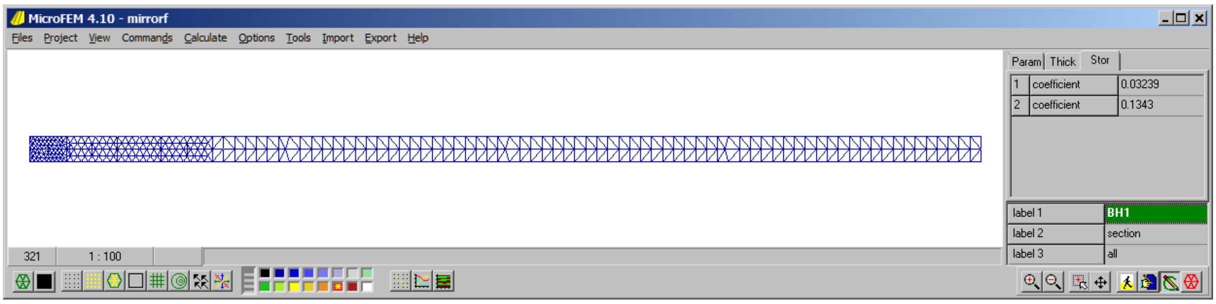


Drawdown contours (0.02 m) of half of the model, after 0.1 day.

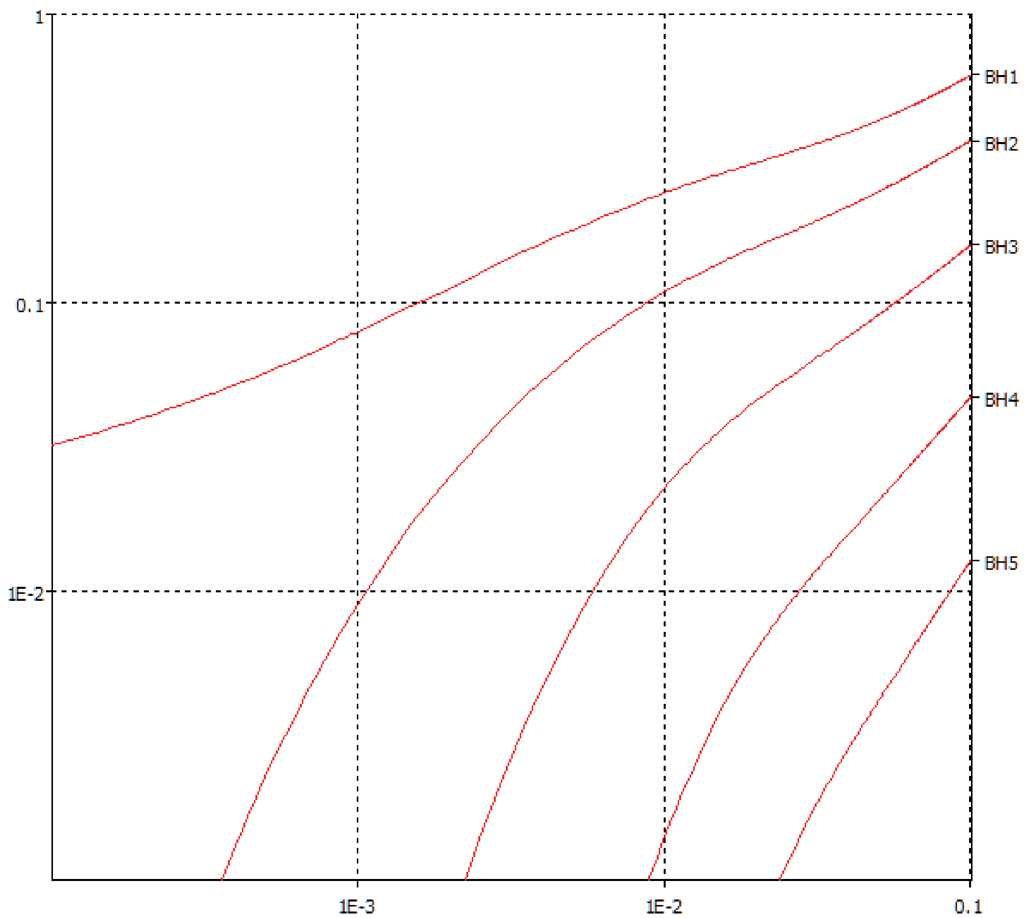
Analysis with MicroFEM

As an independent check on the MLU results, these (analytical model) results are compared with a simple MicroFEM (finite element) model.

The finite element grid consists of only 390 nodes and 572 elements.



The MicroFEM model does not require image wells, while the aquifer thickness reduction as a result of the declining water table can be accounted for. The drawdown curves of the obs. wells computed with MicroFEM and plotted on a log-log scale are very similar to the above MLU results.



Conclusions (March 24, 2018):

The Durham Model Aquifer Pumping test (March 23, 2018) should not be analysed with a Theis model, but with a Boulton model.

MLU can be used to analyse the pumping test, on the condition that a sufficient number of image wells is used, and Jacob's correction is applied to the drawdown measurements.

The obtained hydraulic properties can be found within narrow ranges.

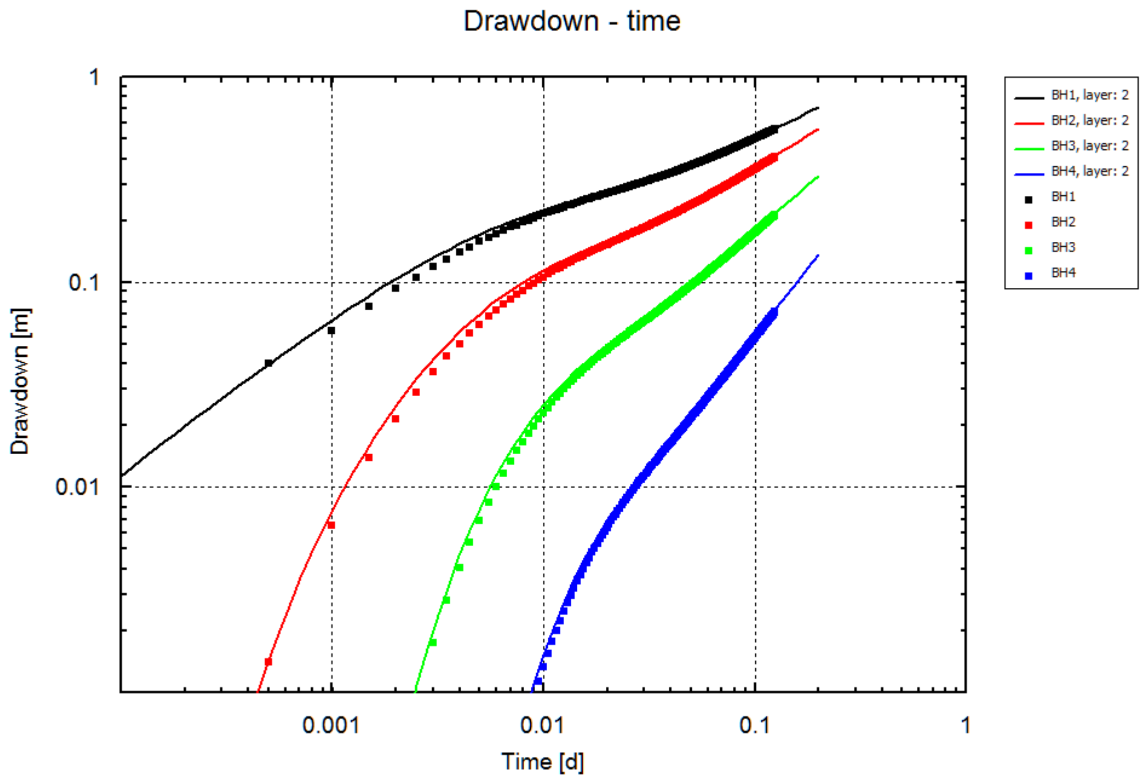
$T = 48\ m^2/d$, $S = 0.032$ and $S_y=0.13$

Step 3: Comparison of models: new corrections

This pumping test analysis could be stopped with the above conclusions. However, now that we have an analytical (MLU) Boulton model and a numerical (MicroFEM) Boulton model it is interesting to do some further analysis and find out:

1) How large is the difference between the models exactly
2) Does the image well solution work properly
3) Does Jacob's correction work properly.

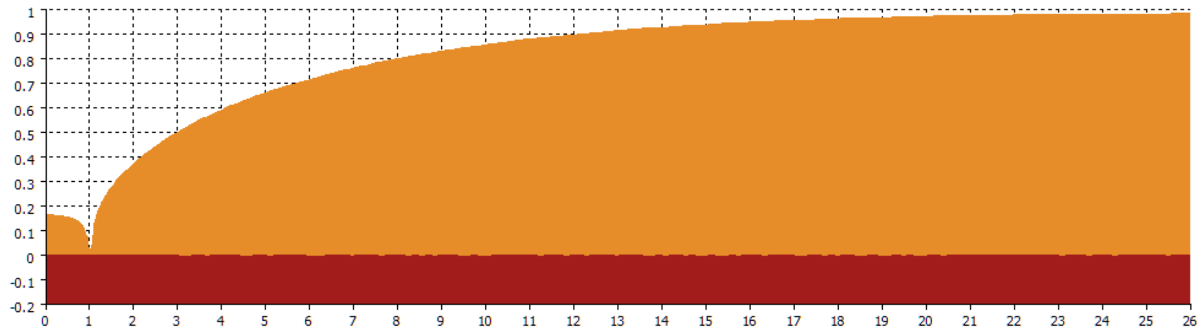
The MLU model and the MicroFEM model have the same hydraulic properties (results of MLU Boulton model):
T1=46.84 m^2/d, c =0.2636 d, S =0.02912, S_y=0.1357
Delay index (1/alpha) = $c.S_y$ = 0.0358
Q_{well}=2.938 m^3/d
The transmissivity in the MicroFEM model is fixed.



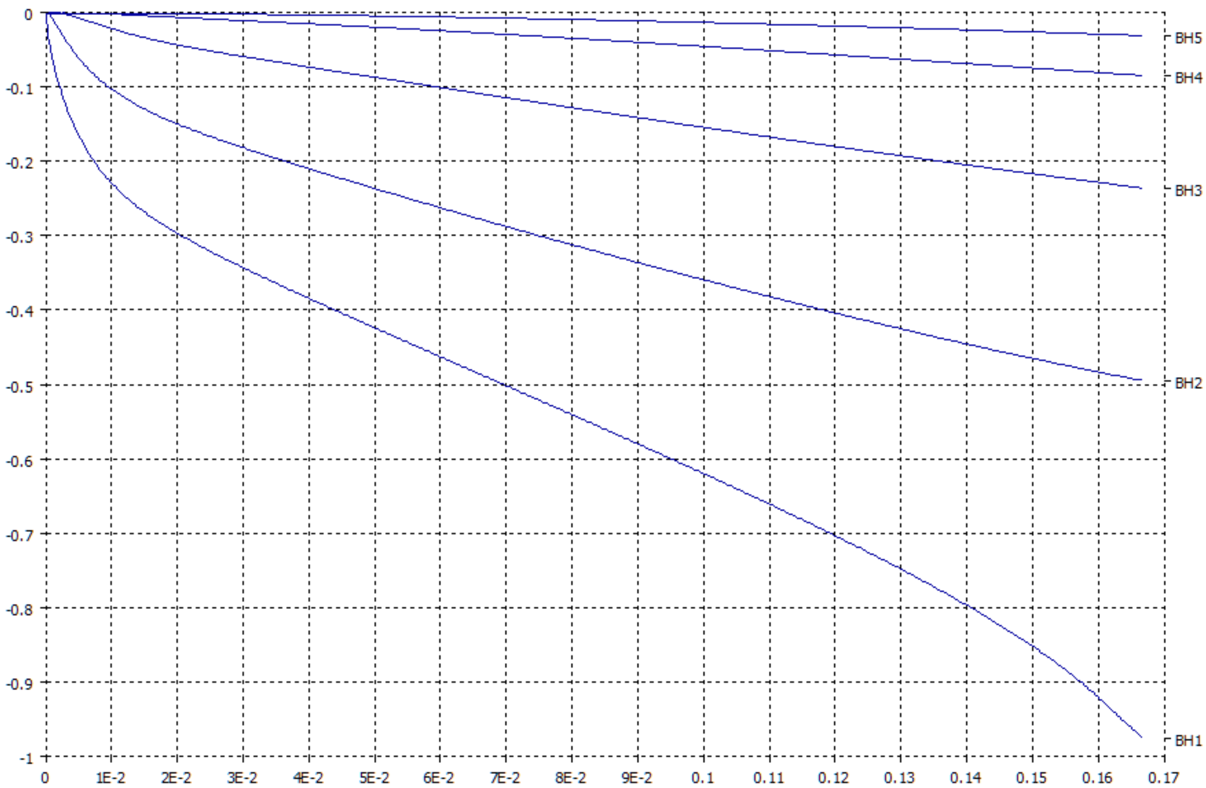
Comparison MicroFEM vs. MLU for a fixed saturated thickness

Results are very similar. MicroFEM drawdowns are somewhat lower (up to 10%) until t is about 0.02 days, maybe because of space or time discretization. Intermediate and late time drawdowns are identical. The image well solution appears to work well.

Now that we know that MLU and MicroFEM produce the same drawdowns, the effect of the reduced saturated thickness is taken into account in the MicroFEM model.

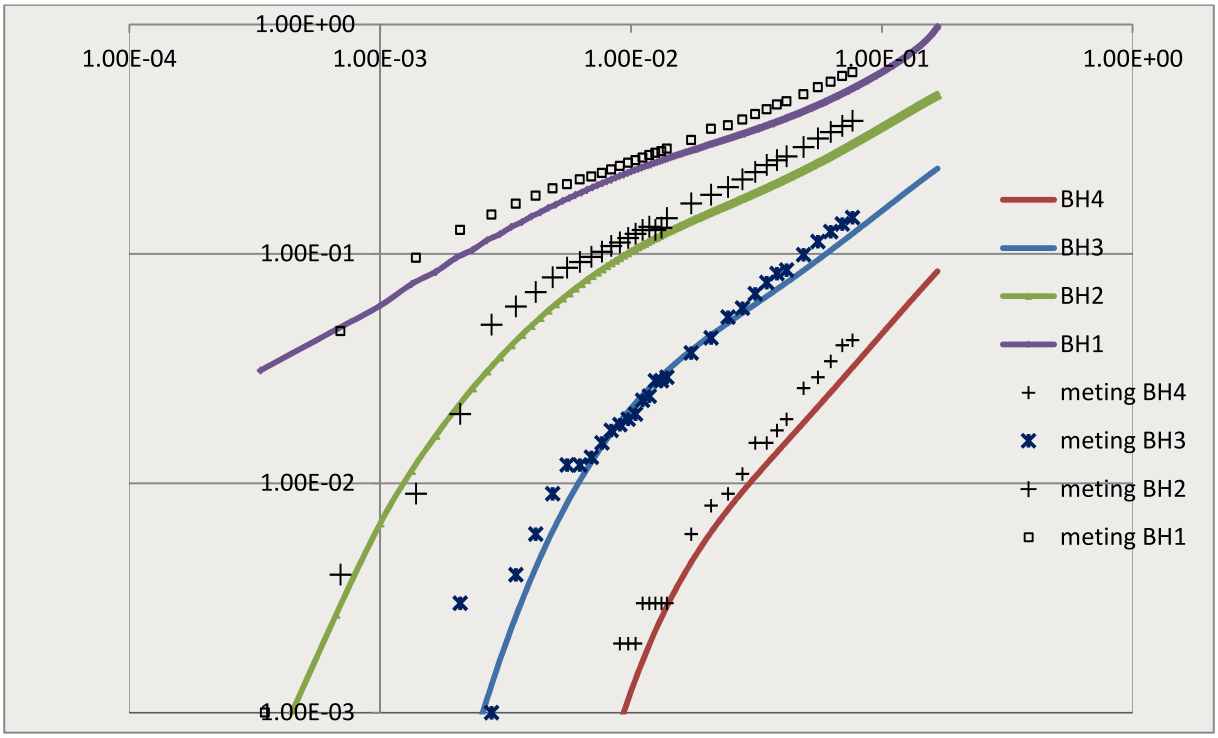


Cross section through the well at t=4 hours

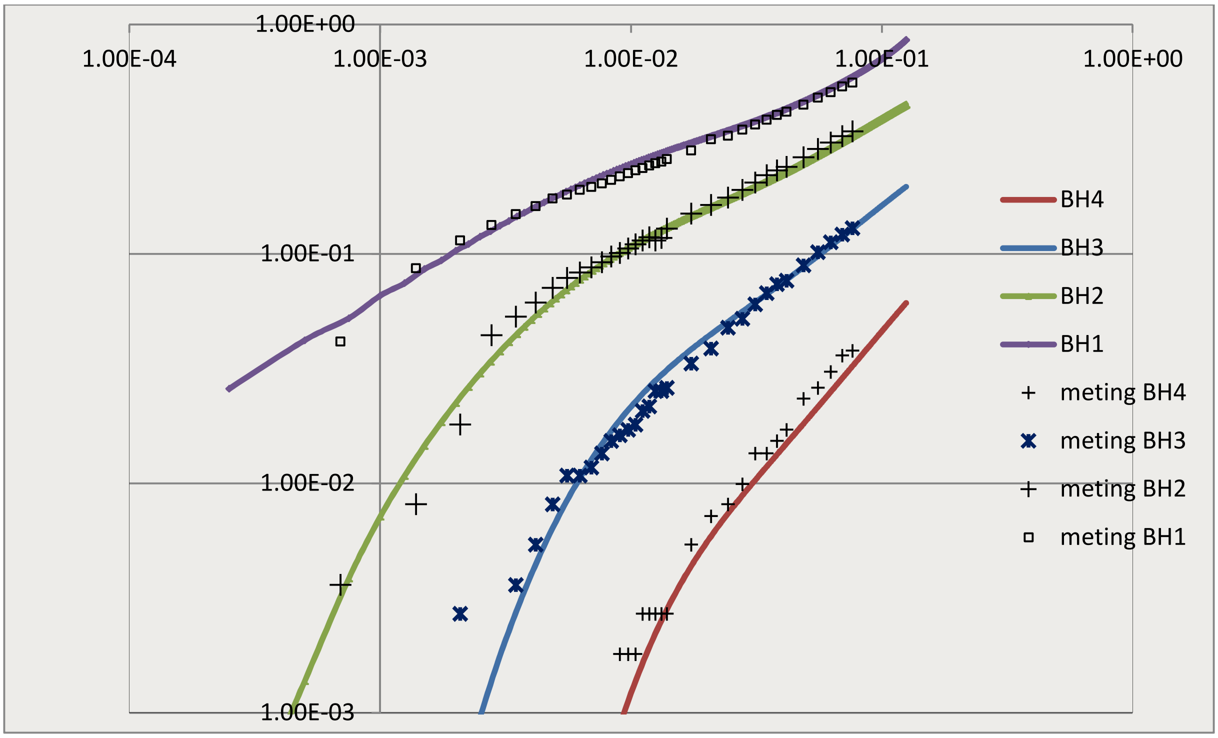


MicroFEM model drawdown in five boreholes. When pumping was not stopped, the aquifer would run dry in BH1 just after 4 hours (t=0.17 d).

These MicroFEM results are compared with the measurements (Excel)



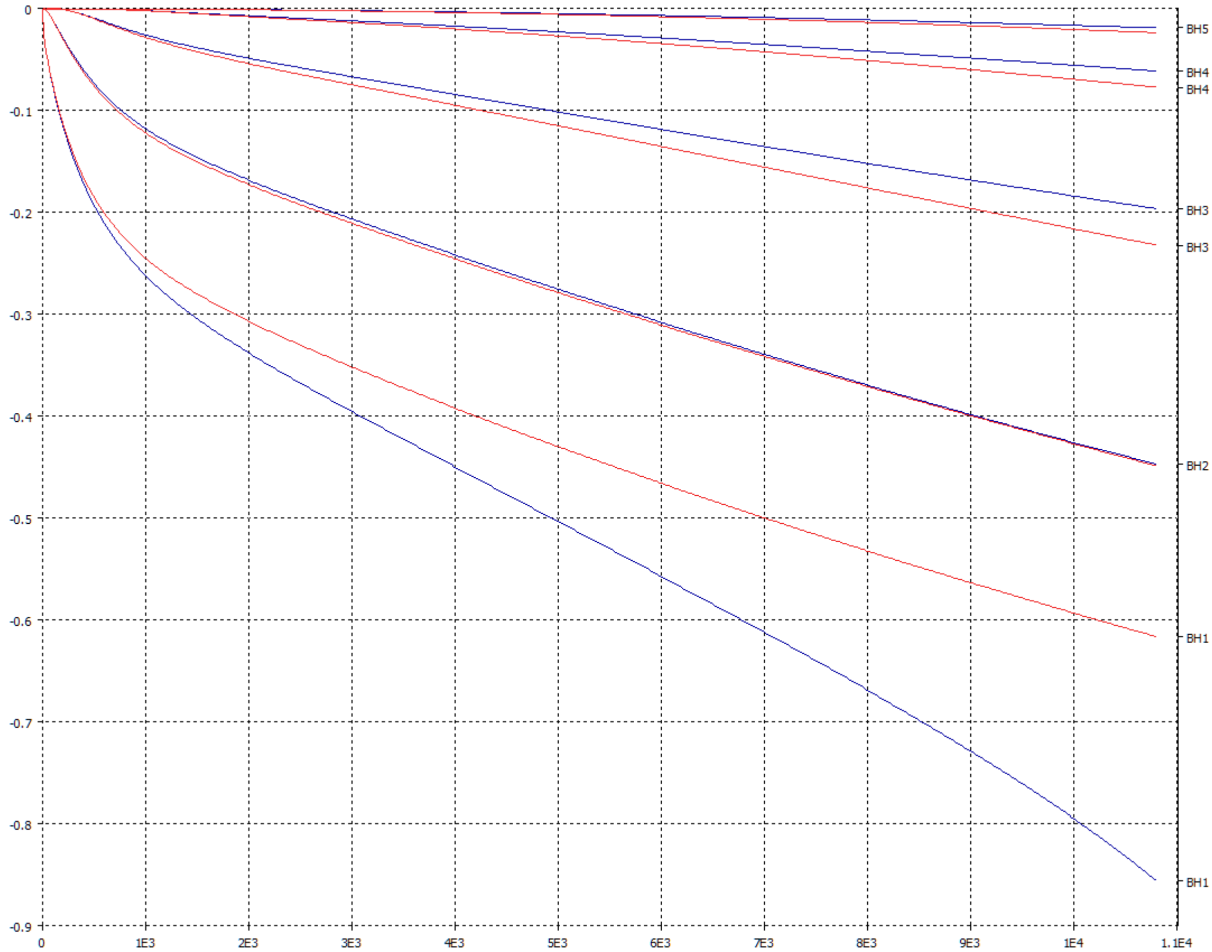
Measured drawdown are about 10% larger than in the MicroFEM model. Apparently the obtained parameters from the MLU model (T and S and S_y) are too high and should be reduced.



MicroFEM model: T1=42 m2/d, S =0.0262 and Sy=0.122

This means that the Step 2 MLU solution leaves us with a 10% error in the results and the only cause of this error must be Jacob's correction.

To obtain an accurate value for the correction two MicroFEM models are run, one with reducing transmissivity (blue curves) and one with a fixed transmissivity (red curves). T1=42 m^2/d, S =0.0262 and S_y=0.122.



MicroFEM drawdown with (blue) and without (red) accounting for a declining water table.

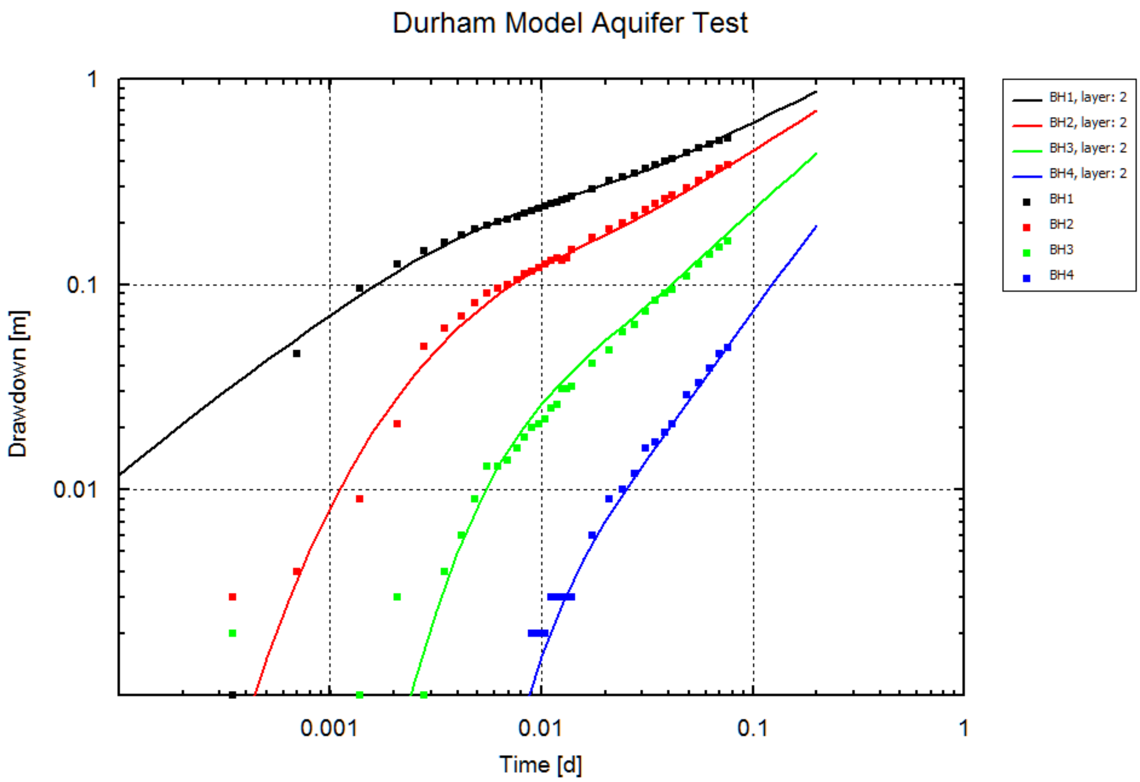
Drawdown in the non-linear model is higher in BH1 (pumping well), about the same in BH2 (2 m from the well) and lower in BH3, BH4 and BH5.

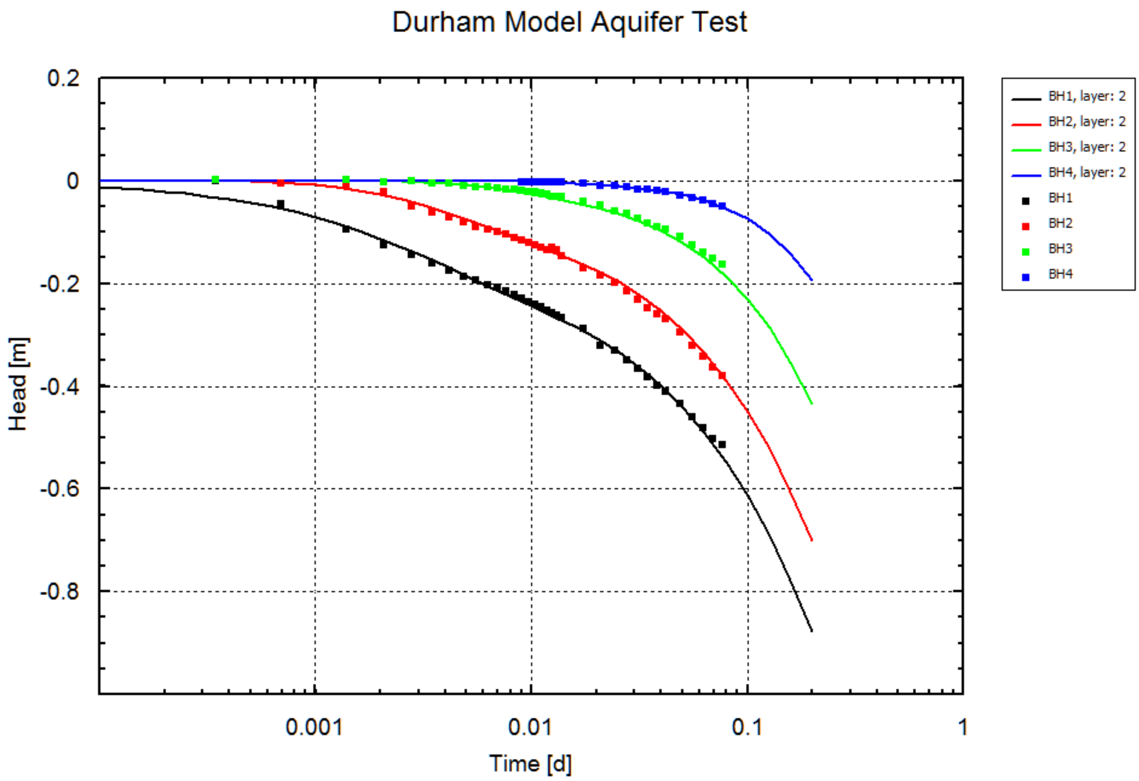
Now that we know the precise effects of the reducing transmissivity for each observation borehole during the pumping test, we can correct the measurements to obtain the values applicable for a constant transmissivity model.

Time (s)	Drawdown Correction (m)					Corrected drawdown (m)				
	BH5	BH4	BH3	BH2	BH1	BH5	BH4	BH3	BH2	BH1
30	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.002	-0.003	0.001
60	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.004	0.046
120	0.000	0.000	0.000	0.000	-0.001	0.001	0.000	-0.001	0.009	0.095
180	0.000	0.000	0.000	0.001	-0.002	0.001	0.000	0.003	0.021	0.125
240	0.000	0.000	0.000	0.001	-0.003	0.001	0.000	0.001	0.050	0.145
300	0.000	0.000	0.000	0.002	-0.004	0.001	0.000	0.004	0.061	0.161
360	0.000	0.000	0.000	0.002	-0.005	0.001	0.000	0.006	0.070	0.174
420	0.000	0.000	0.000	0.002	-0.006	0.001	0.000	0.009	0.081	0.187
480	0.000	0.000	0.001	0.003	-0.007	0.001	0.000	0.013	0.090	0.194
540	0.000	0.000	0.001	0.003	-0.009	0.001	0.000	0.013	0.095	0.202
600	0.000	0.000	0.001	0.003	-0.010	0.001	0.000	0.014	0.100	0.207
660	0.000	0.000	0.001	0.003	-0.011	0.000	0.000	0.016	0.105	0.214
720	0.000	0.000	0.001	0.004	-0.012	0.001	0.000	0.018	0.112	0.221
780	0.000	0.000	0.002	0.004	-0.013	0.000	0.002	0.020	0.116	0.228
840	0.000	0.000	0.002	0.004	-0.014	0.001	0.002	0.021	0.121	0.235
900	0.000	0.000	0.002	0.004	-0.015	0.001	0.002	0.022	0.126	0.241
960	0.000	0.000	0.002	0.004	-0.016	0.000	0.003	0.025	0.131	0.246
1020	0.000	0.000	0.002	0.004	-0.017	0.002	0.003	0.026	0.135	0.252
1080	0.000	0.000	0.003	0.004	-0.018	0.001	0.003	0.031	0.131	0.257
1140	0.000	0.000	0.003	0.004	-0.019	0.001	0.003	0.031	0.134	0.261
1200	0.000	0.000	0.003	0.004	-0.020	0.001	0.003	0.032	0.147	0.267
1500	0.000	0.000	0.004	0.004	-0.024	0.000	0.006	0.041	0.170	0.289
1800	0.000	0.001	0.005	0.004	-0.028	0.001	0.009	0.048	0.185	0.322
2100	0.000	0.001	0.006	0.004	-0.032	0.000	0.010	0.059	0.199	0.331
2400	0.000	0.001	0.006	0.004	-0.036	0.001	0.012	0.064	0.215	0.349
2700	0.000	0.001	0.007	0.004	-0.040	0.001	0.016	0.074	0.231	0.366
3000	0.000	0.002	0.008	0.004	-0.044	0.001	0.017	0.083	0.248	0.382
3300	0.000	0.002	0.009	0.004	-0.048	0.001	0.019	0.091	0.260	0.399
3600	0.000	0.002	0.009	0.004	-0.052	0.001	0.021	0.094	0.270	0.410
4200	0.001	0.003	0.011	0.004	-0.061	0.002	0.029	0.110	0.296	0.435
4800	0.001	0.004	0.013	0.003	-0.070	0.001	0.033	0.126	0.321	0.461
5400	0.001	0.005	0.015	0.003	-0.080	0.001	0.039	0.140	0.342	0.482
6000	0.001	0.006	0.017	0.003	-0.092	0.001	0.046	0.152	0.364	0.503
6600	0.002	0.007	0.019	0.002	-0.104	0.002	0.049	0.163	0.381	0.514

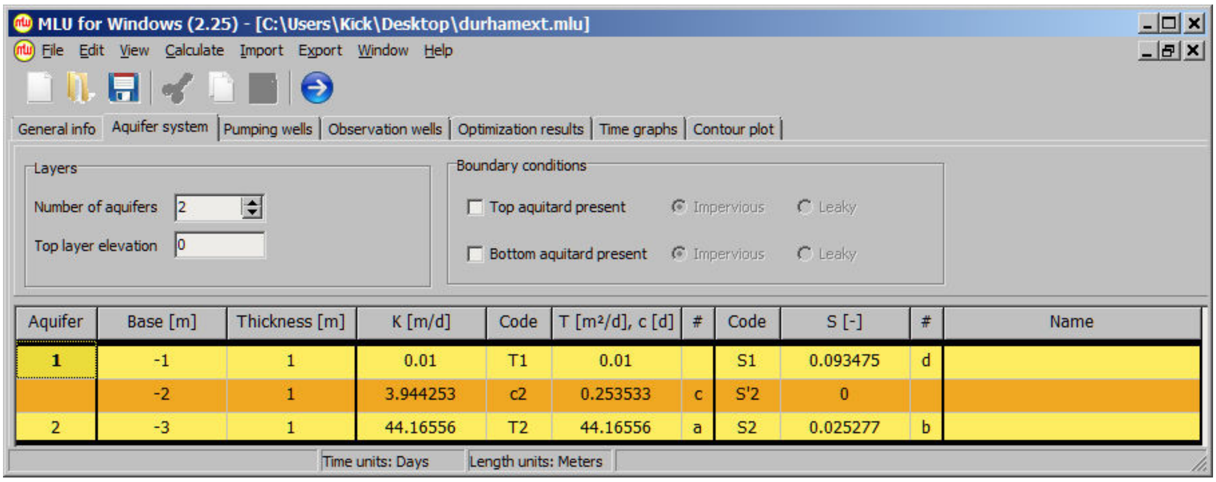
Step 4: Results of final MLU Analysis

The corrected drawdowns can be analysed using MLU.





Measured and computed heads on a log-log and log-linear scale



Aquifer	Base [m]	Thickness [m]	K [m/d]	Code	T [m²/d], c [d]	#	Code	S [-]	#	Name
1	-1	1	0.01	T1	0.01		S1	0.093475	d	
	-2	1	3.944253	c2	0.253533	c	S'2	0		
2	-3	1	44.16556	T2	44.16556	a	S2	0.025277	b	

M L U A Q U I F E R T E S T A N A L Y S I S

THE CALCULATED LEAST SQUARES SOLUTION

Parameter	value	+	Standard deviation	
T 2	4.233E+01	+	5.014E-01	(1 %)
c 2	2.555E-01	+	9.729E-03	(4 %)
S 1	9.305E-02	+	1.508E-03	(2 %)
S 2	2.609E-02	+	8.615E-04	(3 %)

Residual sum of squares is 0.0054
Improvement last iteration 1.2E-11
Condition number 64.2

Correlation matrix (%)

T 2	100			
c 2	11	100		
S 1	-42	19	100	
S 2	-5	85	-8	100

Conclusions

Four parameters can be obtained within small ranges using MLU and a Boulton model: T = 42 m^2/d = 0.00049 m^2/s, S = 0.026 and S_y=0.093
Boulton delay index = c * S = 0.024 d.
Sum of Squared Errors = SSE = 0.0054 m^2

The correction for the reduced aquifer thickness as proposed by Jacob (K&dR, p.101) leads to erroneous results.

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